Principles of Lasers

Fifth Edition

Orazio Svelto



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To my wife Rosanna and to my sons Cesare and Giuseppe

Preface

This book is motivated by the very favorable reception given to the previous editions as well as by the considerable range of new developments in the laser field since the publication of the third edition in 1989. These new developments include, among others, Quantum-Well and Multiple-Quantum Well lasers, diode-pumped solid-state lasers, new concepts for both stable and unstable resonators, femtosecond lasers, ultra-high-brightness lasers etc. The basic aim of the book has remained the same, namely to provide a *broad* and *unified description* of laser behavior at the simplest level which is compatible with a correct physical understanding. The book is therefore intended as a text-book for a senior-level or first-year graduate course and/or as a reference book.

This edition corrects several errors introduced in the previous edition. The most relevant *additions* or *changes* to since the third edition can be summarized as follows:

- 1. A much-more detailed description of Amplified Spontaneous Emission has been given [Chapt. 2] and a novel simplified treatment of this phenomenon both for homogeneous or inhomogeneous lines has been introduced [Appendix C].
- 2. A major fraction of a chapter [Chapt. 3] is dedicated to the interaction of radiation with semiconductor media, either in a bulk form or in a quantum-confined structure (quantum-well, quantum-wire and quantum dot).
- 3. A modern theory of stable and unstable resonators is introduced, where a more extensive use is made of the ABCD matrix formalism and where the most recent topics of dynamically stable resonators as well as unstable resonators, with mirrors having Gaussian or super-Gaussian transverse reflectivity profiles, are considered [Chapt. 5].
- 4. Diode-pumping of solid-state lasers, both in longitudinal and transverse pumping configurations, are introduced in a unified way and a comparison is made with corresponding lamp-pumping configurations [Chapt. 6].
- 5. Spatially-dependent rate equations are introduced for both four-level and quasi-threelevel lasers and their implications, for longitudinal and transverse pumping, are also discussed [Chapt. 7].

- 6. Laser mode-locking is considered at much greater length to account for e.g. new mode-locking methods, such as Kerr-lens mode-locking. The effects produced by second-order and third-order dispersion of the laser cavity and the problem of dispersion compensation to achieve the shortest pulse-durations are also discussed at some length [Chapt. 8].
- 7. New tunable solid-state lasers, such as Ti: sapphire and Cr: LISAF, as well as new rare-earth lasers such as Yb³⁺, Er³⁺, and Ho³⁺ are also considered in detail [Chapt. 9].
- 8. Semiconductor lasers and their performance are discussed at much greater length [Chapt. 9].
- 9. The divergence properties of a multimode laser beam as well as its propagation through an optical system are considered in terms of the M^2 -factor and in terms of the embedded Gaussian beam [Chapt. 11 and 12].
- 10. The production of ultra-high peak intensity laser beams by the technique of chirped-pulse-amplification and the related techniques of pulse expansion and pulse compression are also considered in detail [Chapt. 12].

The book also contains numerous, thoroughly developed, examples, as well as many tables and appendixes. The examples either refer to real situations, as found in the literature or encountered through my own laboratory experience, or describe a significative advance in a particular topic. The tables provide data on optical, spectroscopic and nonlinear-optical properties of laser materials, the data being useful for developing a more quantitative context as well as for solving the problems. The appendixes are introduced to consider some specific topics in more mathematical detail. A great deal of effort has also been devoted to the *logical organization* of the book so as to make its content more accessible.

The *basic philosophy* of the book is to resort, wherever appropriate, to an intuitive picture rather than to a detailed mathematical description of the phenomena under consideration. Simple mathematical descriptions, when useful for a better understanding of the physical picture, are included in the text while the discussion of more elaborate analytical models is deferred to the appendixes. The *basic organization* starts from the observation that a laser can be considered to consists of three elements, namely the active medium, the resonator, and the pumping system. Accordingly, after an introductory chapter, Chapters 2–3, 4–5 and 6 describe the most relevant features of these elements, separately. With the combined knowledge about these constituent elements, chapters 7 and 8 then allow a discussion of continuos-wave and transient laser behavior, respectively. Chapters 9 and 10 then describe the most relevant types of laser exploiting high-density and low-density media, respectively. Lastly, chapters 11 and 12 consider a laser beam from the user's view-point examining the properties of the output beam as well as some relevant laser beam transformations, such as amplification, frequency conversion, pulse expansion or compression.

With so many topics, examples, tables and appendixes, it is clear that the entire content of the book could not be covered in only a one semester-course. However the organization of the book allows several different learning paths. For instance, one may be more interested in learning the *Principles of Laser Physics*. The emphasis of the study should then be mostly concentrated on the first section of the book [Chapt. 1–5 and Chapt. 7–8]. If, on the other hand, the reader is more interested in the *Principles of Laser Engineering*, effort should mostly be concentrated on the second part of the book Chap. 6 and 9–12. The *level of understanding*

Preface

of a given topic may also be suitably *modulated* by e.g. considering, in more or less detail, the numerous examples, which often represent an extension of a given topic, as well as the numerous appendixes.

Writing a book, albeit a satisfying cultural experience, represents a heavy intellectual and physical effort. This effort has, however, been gladly sustained in the hope that this edition can serve the pressing need for a general introductory course to the laser field.

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Milano

Orazio Svelto

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